

Study of temperature distribution in a Stirling engine regenerator



R. Gheith^{a,*}, F. Aloui^{b,c}, S. Ben Nasrallah^a

^a Université de Monastir, École Nationale d'Ingénieurs de Monastir, Laboratoire LESTE, Avenue Ibn El Jazzar 5019, Monastir, Tunisia

^b GEPEA UMR-CNRS 6144, École des Mines de Nantes, Département Systèmes Énergétiques et Environnement, 4 rue Alfred Kastler, BP20722, 44307 Nantes Cedex 03, France

^c Université de Valenciennes et du Hainaut-Cambrésis, ENSIAME Lab. TEMPO – EA 4542, DF2T, Le Mont Houy, 59313 Valenciennes Cedex 9, France

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ABSTRACT

A gamma Stirling engine is studied in this paper. A special care was accorded to the instrumentation of this engine and especially the instrumentation of the regenerator. A preliminary set of experimental measurement reveals a difference of temperature between both regenerator sides. A second set of experiments was proposed to detect the influence of this phenomenon on Stirling engine performances. The asymmetry of heat transfer inside the Stirling engine regenerator's is one of the important phenomenons which consume a part of the produced energy. Two experiments are made to find out the causes of this asymmetry. In order to know the influence of the different operation parameters on this new phenomenon the experimental design method is adopted. The experimental design is an alternative to identify the parameters sets allowing optimal Stirling engine performances. A central composite rotatable design was adopted for minimizing the asymmetry of temperature between both regenerator sides and maximizes the engine brake power. The selected four independent parameters are: heating temperature (300 °C–500 °C), initial filling pressure (3 bar–8 bar), cooling water flow rate (0.2 l/m–3 l/min) and operation time (4–20 min after study regime). The four adopted factors are experimentally varied. The results show that the heating temperature is the most significant factor for the studied phenomenon. The major damages caused by this phenomenon will be presented too.

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1. Introduction

The optimization of Stirling engine performances is one of the priorities of the related researchers in the past several years. So, many experimental and analytic studies were done to understand phenomena of heat transfer [1], irreversibility [2], imperfect regeneration and flow friction [3] in Stirling engine. The performance of such engines is very sensible to its geometrical [4–7] and operation conditions [6–10].

The temperature distribution in Stirling engine is different from a compartment to another. The most important variations are observed in the regenerator (porous media, complicated heat exchange, heated and cooled at the same time). Temperature distribution is a determinant parameter to the regenerator effectiveness. The regenerator is also the seat of an important part of thermal losses recorded in a Stirling engine [11]. The most considered losses are lost by pressure drop, power lost by external and internal conduction and the regenerator ineffectiveness. Tlili et al. [11] showed that the power loss observed in the Stirling regenerator are about 86% of the total engine losses.

Popescu et al. [2] propose a finite time thermodynamic optimization of a Stirling engine based on endo and exo-irreversible cycle. They conclude that the internal irreversibilities are due to the internal conductance of the plant and to the non-adiabatic regenerator (drops of 30% of engine performances). The external irreversibilities are due to the linear interactions between the heat sources and the working fluids at finite temperature gaps.

The regenerator parameters have been the subject of several experimental and analytical investigations. The regenerator effectiveness was one of the decision variables used in the Multi-objective evolutionary algorithms developed by Ahmadi et al. [12]. They proposed an optimal Pareto frontier in objectives spaces. Chen et al. [13] built a prototype helium-charged twin-power piston c-type Stirling engine. Some of its geometrical and operational parameters were investigated by a numerical model. They found that regeneration effectiveness had the most prominent effect on efficiency, while engine speed had the greatest effect on the engine power within the range of the engine speed. Cheng et al. [14] developed and tested a beta-type 300-W Stirling engine. The engine parameters are insured in a non-ideal adiabatic model to predict its performances. Using a 120 wire mesh regenerator, the shaft power of the engine reaches 390 W at 1400 rpm with 1.21-kW input heat transfer rate (32.2% thermal efficiency). Kato and baba

* Corresponding author.

E-mail address: ramla2gheith@yahoo.fr (R. Gheith).

Nomenclature

b_0	constant coefficient	W	diagonal matrix
b_j	linear coefficients	X	level matrix of independent variable
b_{ii}	quadratic coefficients	x_i	coded values of the factor or regression coefficient
b_{ij}	second order interaction coefficients	x_{cp}	independent variable real value at the center point
$\dot{m}$	cooling water flowrate	$\bar{X}^*$	average value of x_i
N	total number of experiments	Δx_i	the step change of the real value corresponding to a variation of a unit for the dimensionless value of the variable i
P_i	initial filling pressure	Y	vector of observation
Q	degrees of freedom	Y_{mod}	predicted response
T	temperature	Y^*	average value of Y
T_h	heating temperature		
T_{op}	operation time		
ΔT_{exp}	experimental difference of temperature between both regenerator sides		
ΔT_{pr}	difference of temperature between both regenerator side obtained from the predicted model		
U_i	experimental response		
$U_i(0)$	value of U_i in the middle		
ΔU_i	maximum variation of U_i		

Greeks symbols

α	critical value of Pareto chart
β	regression vector coefficients
μ	mean value of the x variables
σ	standard deviation

[15] proposed an experimental method for regenerator evaluation. They concluded that the efficiency of the regenerator depends on the difference of temperature of the working fluid on the hot side and on the cold side and the pressure fluctuation. Chen et al. [16] noticed that the operating temperature ratio dependent on the thermal resistance of heating and cooling spaces, the regenerator effectiveness, the working fluid mass and the engine speed. Glushenkov et al. [17] proposed a thermodynamic analysis based on a heated regenerator. The developed thermodynamic model incorporates non-ideal features of the cycle, such as specific regenerator efficiency, dead volumes and other geometrical parameters. The model shows that the energy efficiency is highly sensitive to regenerator performance. Ahmadi et al. [18], optimize a solar-powered high temperature differential Stirling engine with multiple criteria. The volumetric ratio and the regenerator effectiveness effects on the final selected solution were studied and it was found that a final optimal solution with higher values of objectives (thermal efficiency and output power) could be achieved if these parameters were increased. Kato et al. [19] presented an empirical estimation of Stirling engine regenerator efficiency. The regenerator efficiency is calculated as a function of the temperature fluctuation in the regenerator and the temperature fluctuation of cold side working fluid. They showed that these two parameters are positively correlated with the regenerator loss. Cheng and Yu [20] proposed a more realist model to study beta-type Stirling engines. Their model took into consideration the temperature difference between the working gas and the external heat source, pressure drop in the regenerator, and regenerator effectiveness. Formosa and Despesse [21] included dead volume, imperfect regenerator, and external and internal thermal transfers in their analytical model that could be used to analyze Stirling. Chen et al. [22] experimentally studied the effects of several regenerator parameters on the overall performance of Stirling engine. They incorporated a moving regenerator in their engine. The parameters investigated include regenerator matrix material, matrices arrangement, matrix wire diameter, and fill factor. Anderson et al. [23] concluded that the convective heat transfer is the dominate heat transfer mechanism between regenerator matrices and working gas. Regenerator heat transfer rate is a very crucial factor to regenerator effectiveness especially at high engine speed when the time for heat transfer is very short in each cycle. Therefore, a regenerator with low heat transfer rate will perform poorly at high engine speed even though it has large heat capacity. Gheith

et al. [24] experimented a gamma type Stirling engine with different regenerator materials (Cooper, Aluminum, Stainless Steel and Monel 400). The obtained experimental results provide guidance to Stirling engine enhancement and selection of the appropriate regenerator constituting material. The Monel 400 and the stainless steel regenerators present an acceptable thermal efficiency and do not oxidize. Tavakolpour et al. [25] studied numerically a LTD (Low Temperature Difference) Stirling engine. They found that the energy losses in the regenerator are due to temperature oscillation and heat conduction caused by the alternating gas flow and steep temperature gradient throughout the regenerator and found that the engine's thermal efficiency resulted from the regenerator efficiency of 1.0 at ideal conditions the is 0.069 whereas for the regenerator efficiency of zero, is about 0.0122. The use of an efficient regenerator increases six times the engine thermal efficiency.

The experimentation of Stirling engine is always made by classical way (univariate study) [25–30]. The investigation by such method requires a higher number of experiments for exploring the whole experimental domain. It is time consuming and does not depict the complete effect of each parameter in the response (Stirling engine performances).

To reduce these disadvantages during experimental studies, especially for study involving several variables factors, the experimental design methodology seems to be adequate to avoid such inconvenient.

The experimental design methodology is not a new technique it was developed by Fisher in 1952 [31] and applied first in agronomy. –This method presents several advantages over the conventional one:

- Decrease the number of test performed to scan the entire field of study.
- Possibility to study a large number of factors at a time.
- Detect single or double interactions between the studied factors, [32].
- Modeling the studied responses.
- Determining optimal results [31].

The aim of this paper is to study a new loss which dissipates a part of Stirling engine performances. Detailed descriptions of the experimental set-up as well as its corresponding metrology are presented.

The first section presents a global characterization of the temperature distribution on the regenerator. A zoom on both regenerator sides shows an important gap of temperature. By performing additional experimental measurements it was possible to determine the causes and the consequences of this phenomenon on the engine performances.

A new way of optimization of Stirling engine performances is proposed in the second section of this paper with consideration of the new observed phenomenon. The experimental design methodology was adopted to detect the optimal parameters minimizing the anomaly of temperature distribution in the regenerator and consequently maximizing the performances of the Stirling engine. For operation factors (heating temperature, initial filling pressure and cooling water flow rates) are varied and their corresponding influence on the studied responses are detected and discussed.

2. Experimental set up

2.1. Gamma Stirling engine

The experimental setup is presented in Fig. 1. It is a gamma Stirling engine, working with a maximal compressed air of 10 bar. The maximal rotation speed is around 600 rpm and provides 500 W of mechanical shaft power. In addition to the expansion and compression spaces, the engine includes three heat exchangers: the regenerator, the cooler, and the heater. The regenerator (Fig. 1) is a porous medium (i.e., made of copper with 90% porosity). The cooler acts as a cold source formed by an open water circuit, and the heater consists of 20 tubes in order to increase the exchange surface between the working gas and the hot source

Table 1
Technical specifications of the Stirling engine.

Geometrical parameters			
Compression space, "c"		Regenerator, "r"	
Diameter	65 mm	Diameter	52 mm
Height	170 mm	Height	153 mm
Expansion space, "e"		Porosity	0.9
Diameter	65 mm	Heater, "h"	
Height	170 mm	Diameter	12 mm
Cooler, "k"		Height	650 mm
Diameter	12 mm	Crank radius	66 mm
Height	900 mm	Compression ratio	3.5
Functioning parameters			
Nominal rotation speed	600 rpm	Working fluid	Air
Hot source temperature	500 °C	Cold source temperature	15 °C
Porosity	0.9	Regenerator material	Stainless steel

(electrical resistance). The operating conditions and the geometrical characteristics of the studied Stirling engine are recapitulated in Table 1.

2.2. Instrumentation

To characterize the Stirling engine, we performed the experiment using different parameters such as the pressure, the temperatures at different positions in the engine, and the cooling water flowrate. Several type-K thermocouples with a diameter of 0.5,

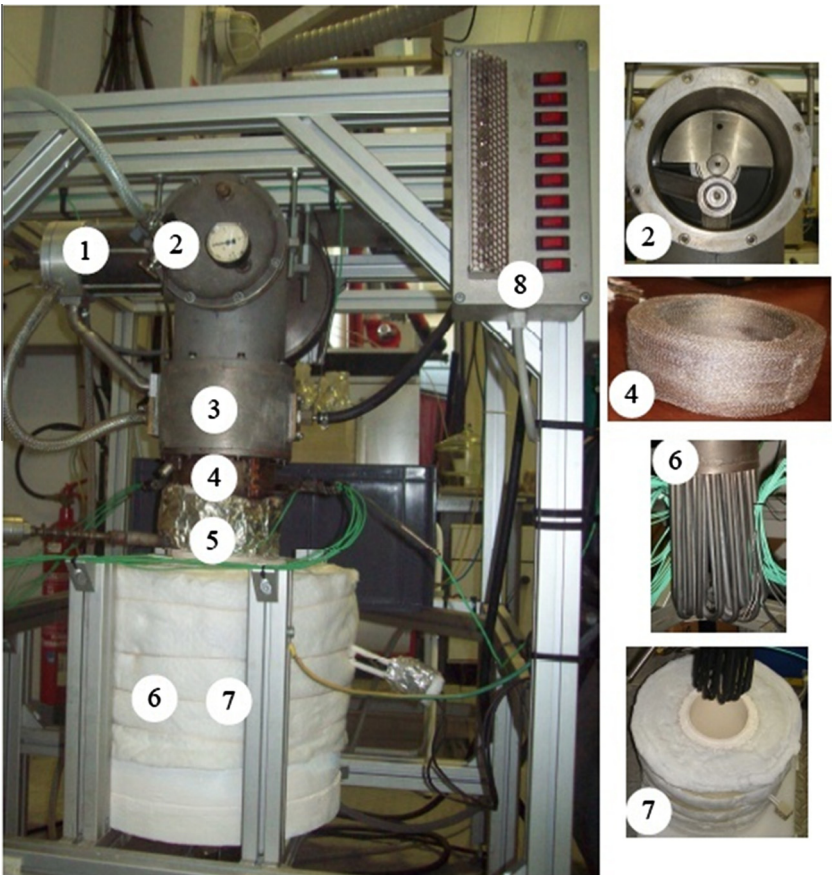


Fig. 1. The Stirling engine set-up: 1 – expansion space, 2 – drive system, 3 – cooler, 4 – regenerator, 5 – compression space, 6 – Heater, 7 – heating system, 8 – dissipation system.

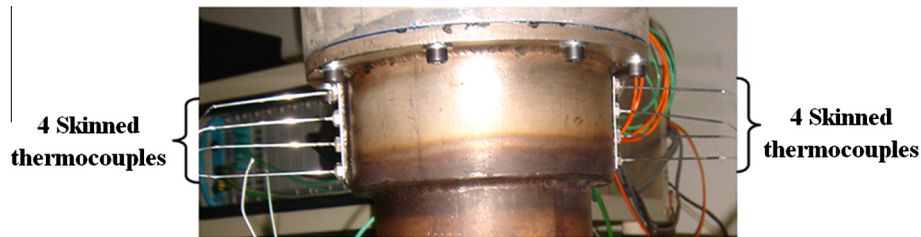


Fig. 2. The regenerator instrumentation.

0.25, and 0.0254 mm were used to measure the temperature at different locations of the engine. Because the interest was to measure the instantaneous temperatures, proper care has been taken regarding the size and the thermal inertia (which should be very low) of the thermocouples. Two thermocouples were mounted upstream of the expansion and compression spaces, respectively. Other two thermocouples were used to measure the inlet and outlet temperatures of the water supplying to the cooler. Eight thermocouples are implemented to measure the temperature of the working gas passing through the symmetrical side of the regenerator matrix. The thermocouples were skinned up to 1 mm in the regenerator matrix without touching the material (Fig. 2). The brake power produced by the Stirling engine was transmitted by a transmission belt to an alternator. The alternator was mounted on a balance plate that has been used to determine the torque transmitted from the transmission belt to the alternator. A flow-meter localized upstream of the entrance of the engine in the water-cooling circuit was used to determine the flow rate of the cooler. A crank angle transducer was connected to the engine shaft. Pressure transducers were installed in the compression space and after the expansion space. Their frequency response was chosen for a wide range to ensure good instantaneous measurements of the pressures inside the working spaces.

Acquisition data were recorded using a fast acquisitions system (ADwin Pro-II, Jäger Computergesteuerte Messtechnik GmbH, Lorsch, Germany). The ADwin Pro-II is an external processing system with modular expansion options. Our system includes eight modules in parallels. The first two modules were used to the acquisition of the thermocouple signals. Each module is formed of eight multiplexes input with 18 bits of precision. Hence, the maximum uncertainties on the temperature measurements do not exceed 0.002 °C. The transducers of the pressure in each of the two space rooms, the force and the flowrate, are linked to two analog input modules. Each one of these modules has eight fast analog-to-digital converter of 18 bits and eight differential inputs. The digital input/output module provides 32 programmable digital input and output channels at transistor–transistor logic levels. All these modules are controlled by the processor module. It has an internal memory of 768 KB and has an external trigger input that enables the processor module to recognize an external signal. A crank angle transducer is located in the end of the Stirling engine shaft. On every crankshaft degree, this crank angle transducer sends a transistor–transistor logic external signal to the processor module, which allows the acquisition of all signals by all used modules. For every cycle, 360 acquisitions are done, which correspond to 360° of the crankshaft. For every run, 100 cycles are recorded. The acquisition duration depends on the Stirling engine revolutions per minute (rpm) for the entire 36,000 acquired measurement points and for each signal.

3. Global study of the regenerator distribution of temperature

A Stirling engine regenerator is crossed twice by the working fluid during each Stirling cycle. Assume that the direction of the

working fluid flow is from the cooler to the heater through the regenerator (porous media). During the first phase of the cycle, the regenerator porous matrixes discharge its stored heat to the working fluid (preheating the working fluid). In the opposite direction of flow (the working fluid flowing from the heater to the cooler), the porous matrix is heated by the working fluid (pre-cooling of the working fluid).

Thanks to the adequate thermocouples (very small diameters) placed on both regenerator sides, the instantaneous temperatures of the working fluid at different positions of the regenerator are recorded and represented in Fig. 3. It can be seen that all detected temperatures present periodic evolution with different mean value and amplitude. The temperature distribution in both sides of the regenerator matrix is not identical. Side 1 of the regenerator matrix is warmer compared to side 2. This observation is confirmed by the axial velocity recorded on both regenerator sides. This parameter was calculated using the shift phase between T_{R1} and T_{R4} for side 1 and between T_{R5} and T_{R8} for side 2. The axial velocity of the working gas in the regenerator side 1 is about 0.23 m/s. However, it is about 0.46 m/s in the side 2. The axial velocity of the working fluid in the side 2 of the regenerator is twice than that of side 1. Hence, the heat exchange duration is more important in side 2. The slowest working fluid (side 2), spends more time in the regenerator and therefore has more time to interact with the porous matrix and absorb more calories. It is clear that the heat transfer in both regenerator sides is asymmetrical.

The experimental working gas velocities in both regenerator sides were calculated using the shift phase between T_{R1} and T_{R4} for side 1 and between T_{R5} and T_{R8} for side 2. The axial velocity of the working gas in the regenerator side 1 is about 0.23 m/s. However, it is about 0.46 m/s in the side 2. The axial velocity of the working fluid in the side 2 of the regenerator is twice than that of side 1. Hence, the heat exchange duration is more important in side 2. In fact, the working fluid spends more time in side 1 absorbing more energy. It is clear that the heat transfer in both regenerator sides is asymmetrical.

The heating temperature and the initial filling pressure were varied and the difference of temperature between two thermocouples placed on opposed regenerator side but at the same level are recorded. The difference of temperature between both regenerator sides increase with the heating temperature. It has an average value of 37 °C for a heating temperature of 500 °C (Fig. 4). The initial filling pressure does not have an important influence on the studied phenomenon. For an increase of 5 bar, the difference of temperature increases of only 2 °C for all applied heating temperature.

It is a new lost, which dissipate an important part of thermal energy exchanged in the regenerator. This lost is not considered in the theoretical models encountered in the literature. It depends mainly of the compartment dispositions and design. In the following section the causes and the influence of this lost on the engine performances will be discussed. In addition of internal conduction loss and lost by imperfect regeneration, the lost by dissymmetry

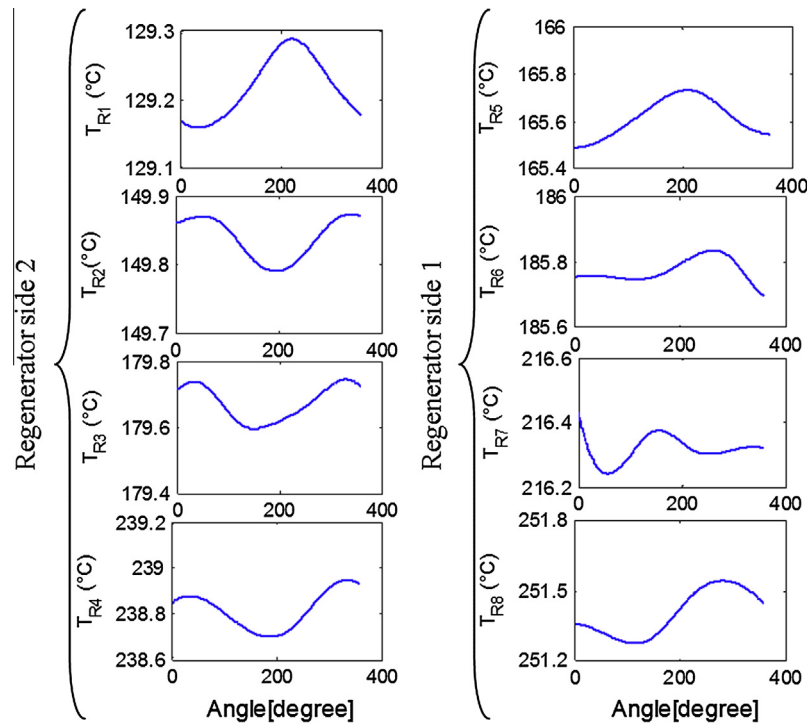


Fig. 3. The working fluid temperature evolution at different position of the regenerator. (Thermocouples T_{R1} to T_{R4} in side 2 & Thermocouples T_{R5} to T_{R8} in side 1). Experimental conditions: $T_h = 500$ °C, $P_i = 8$ bar, $\dot{m} = 8.16$ l/min.

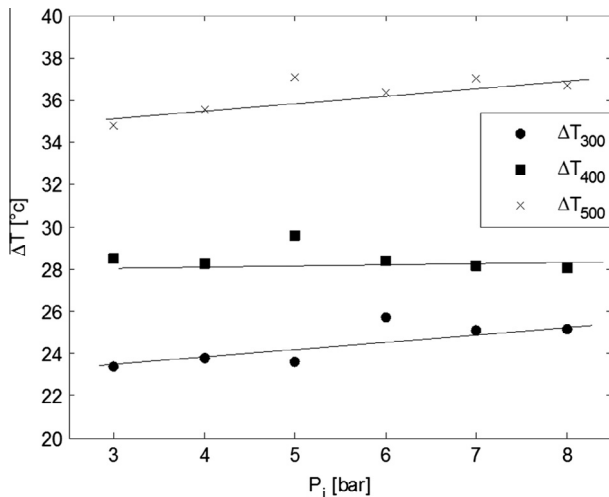


Fig. 4. Evolution of the mean difference of temperature between both regenerator sides function of the initial filling pressure and for different heating temperatures.

regenerator of temperature in the can be added t theoretical model in same case of Stirling engine.

4. Causes of the asymmetry of temperature between both regenerator sides

It is clearly showing in Fig. 5 that the engine design is asymmetrical. The circulation of the working fluid from the compression working space to the cooler is made on the left side of the engine (side 1). The entrance of the cooling water (dark blue) is made on the right side of the Stirling engine (side 2). The side 1 (right side) of the regenerator is more cooled by the water flow rate (heat transfer by conduction through the wall separating the cooler from the working fluid). Indeed at its input to the Stirling engine, the

cooling water is at its minimum temperature, coming to the opposite side of the regenerator, the water becomes hotter.

The previous conclusion is confirmed by a first experience (Fig. 6A). All regenerator temperatures are records when the engine is stopped, and heated at 100 °C. Initially, all thermocouples record nearly the same temperature, a difference is gradually recorded between all thermocouples and especially between both regenerators' sides. After 5300s, the cooling water flow rate is opened; immediately all recorded temperatures decreases. The temperatures T_{R1} and T_{R5} (the nearest to the cooler) are different, the working fluid side 1 (T_{R1} thermocouple) of the regenerator is lower than that of the fluid at the same level but in the opposite side 2 (thermocouple T_{R5}).

The Gamma Stirling engine was heated at a temperature of 300 °C, reaching this temperature the engine was started. All recorded temperatures are represented in Fig. 6B. Initially, the thermocouples present almost the same temperature of the working fluid on both regenerator sides. Then (engine starts), the temperatures have increased over time and eventually stabilize when the engine has reached its quasi-steady operation. Temperatures delivered by thermocouples T_{R1} and T_{R5} , increased with the engine started, but the gap between them has also increased. When the engine start this difference increases from 1 °C to 20 °C on the quasi-steady regime.

5. Experimental design methodology

The experimental design methodology was used to study the effect of four operation parameters on the asymmetry of temperature between both regenerators' sides. The heating temperature, the charge pressure, the cooling water flow rate and the operation time were chosen as independent factors.

The Yate notation (Fig. 7) allows a simple representation of combinations of levels and allows easy calculation of factors effects and interactions. Levels are represented by + and – signs symbolizing the high and the low value of a factor. The relationship between

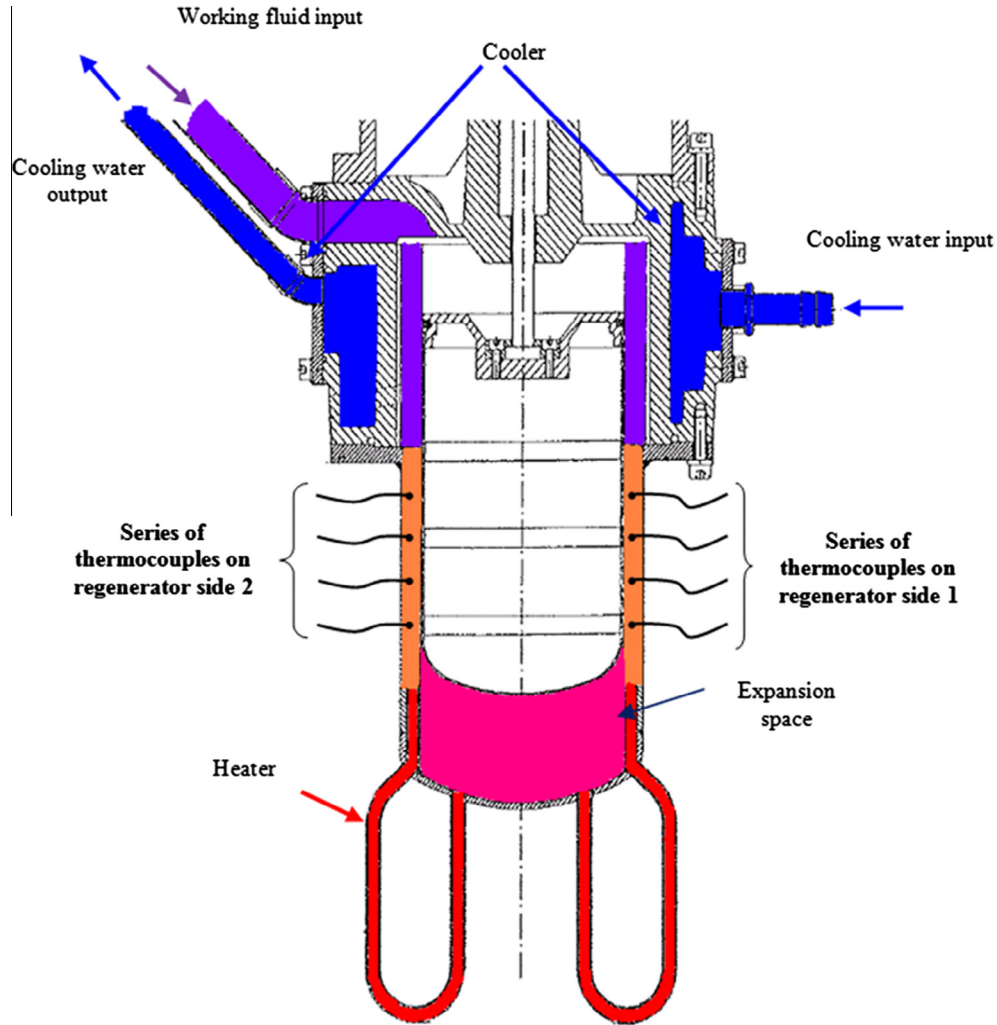


Fig. 5. Part of the cut of the studied Stirling engine.

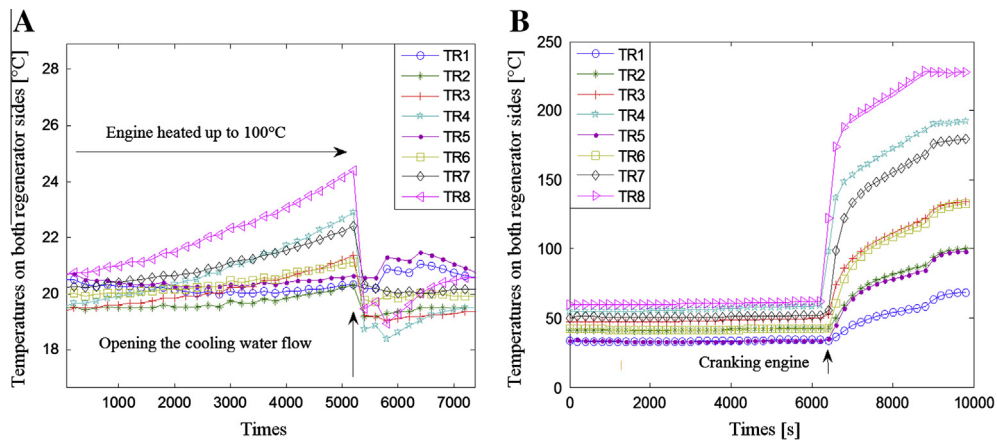


Fig. 6. (A and B) Experiments to determine the existence of the difference of temperature between both regenerator sides.

the coded factors and the encoded ones are calculated by the following formulas:

$$x'_i = \frac{x_i - x_{cp}}{\Delta x_i} \quad (1)$$

$$\Delta x_i = \frac{x_{imax} - x_{imin}}{2} \quad (2)$$

In order to detect the influence of each studied factor, two experimental designs were conducted: the first was a Factorial Centered Design (FCD) for determining the most influencing factors and the second was a Central Composite Rotatable Design (CCRD) for determining the response surfaces model for the more influencing factors and then obtaining the optimal functioning conditions.

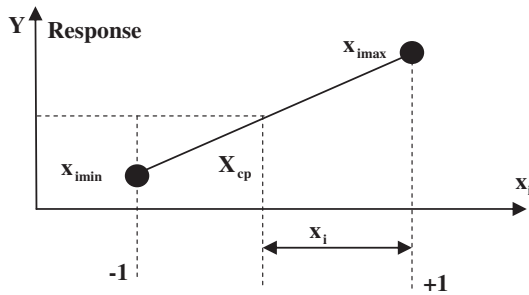


Fig. 7. Yate notation.

The multiple regression analysis was used to obtain a quadratic response surface. The empirical model linking the difference of temperature between both regenerator sides to all studied factors x_i , their interactions $x_i x_j$ and the quadratic terms x_i^2 is writing as:

$$Y_{mod} = b_0 + \sum_{j=1}^4 b_{jj}x_j^2 + \sum_{0 < j < k \leq 4} b_{jk}x_jx_k \quad (3)$$

Eq. (3) can be expressed in the matricial form as follows:

$$Y = XB + \varepsilon \quad (4)$$

Table 2
Coded and actual levels of factors considered for design.

Factors	Levels				
	Lowest (−2)	Lower (−1)	Center (0)	High (+1)	Highest (+2)
(A) Cooling flow rates (l/min)	0.26	2.25	4.21	6.17	8.16
(B) Charge pressure (bar)	3	4.25	5.5	6.75	8
(C) Heating temperature (°C)	300	350	400	450	500
(D) Operating time (s)	4	8	12	16	20

Table 3
Design and the yield response.

Run	Cooling flow rates (l/min)	Charge pressure (bar)	Heating temperature (°C)	Operating time (s)	ΔT_{exp} (°C)
<i>Factorial points</i>					
1	2.25	4.25	350	8	32.28
2	6.17	4.25	350	8	28.10
3	2.25	6.75	350	8	29.29
4	6.17	6.75	350	8	28.34
5	2.25	4.25	450	8	42.90
6	6.17	4.25	450	8	37.70
7	2.25	6.75	450	8	41.90
8	6.17	6.75	450	8	36.68
9	2.25	4.25	350	16	34.28
10	6.17	4.25	350	16	28.10
11	2.25	6.75	350	16	30.29
12	6.17	6.75	350	16	28.34
13	2.25	4.25	450	16	43.90
14	6.17	4.25	450	16	37.70
15	2.25	6.75	450	16	40.03
16	6.17	6.75	450	16	36.68
<i>Centre points</i>					
17	4.21	5.5	400	12	33.97
18	4.21	5.5	400	12	33.01
19	4.21	5.5	400	12	34.53
<i>Axial points</i>					
20	0.26	5.5	400	12	28.50
21	8.16	5.5	400	12	26.14
22	4.21	3	400	12	30.40
23	4.21	8	400	12	29.87
24	4.21	5.5	300	12	24.70
25	4.21	5.5	500	12	47.84
26	4.21	5.5	400	4	32.58
27	4.21	5.5	400	20	35.88

The sum of squared error (SSE) is defined by:

$$SSE = (Y - XB)^T (Y - XB) \quad (5)$$

Taking derivative of SSE with respect to B gives the minimum of SSE from the normal equation:

$$X^T XB = X^T Y \quad (6)$$

So, the estimate, b , of the elements of B is given by:

$$b = (X^T \cdot X)^{-1} X^T Y \quad (7)$$

The total sum of square can be partitioned into two parts, the sum of square due to regression (SSR) and the sum of square unaccounted for by the fitted model (SSE):

$$SSR = (\bar{Y} - Xb)(\bar{Y} - Xb)^T \quad (8)$$

With

$$\bar{Y} = \left(\frac{1}{n} \sum_{i=1}^n Y_i \right) I \quad (9)$$

With consideration of Eq. (4), the total sum of squares is written as:

$$SST = SSR + SSE = \frac{1}{n} (\bar{Y} - Xb)(\bar{Y} - Xb)^T + \frac{1}{n} (Xb - Y)(Xb - Y)^T \quad (10)$$

And then the coefficient of multiple determinations R^2 and the adjusted coefficient of multiple determinations R_{ad}^2 can be determined as follows:

$$R^2 = \frac{SSR}{SST} = 1 - SSE/SST \quad (11)$$

$$R_{ad}^2 = 1 - \frac{SSE/(n - k - 1)}{SST/(n - 1)} \quad (12)$$

The experimental design must choose a set of n combinations of the studied factors. The set of pilot experiment must be different from each other and must belong to a pre-defined domain to explore. The combinations are chosen to efficiently investigate the relationship between the design factors and the response. The experimental plan generated using the Minitab 16 Software, along with the results is depicted in Table 3.

5.1. Determination of the significant parameters on the studied response

In order to determine the exact term (influenced factors, interaction and quadratic terms) of the predicting model, the following approach is used:

The Pareto chart allows detecting the factors and the interactions which are most important to the studied response [33]. In this chart, the length of each bar is proportional to the absolute value of the estimated effect [34]. Effects that cross the vertical line ($y = \alpha$) are statistically significant at 95% confidence level. The value of α is calculated from the t -test [35]:

$$\alpha = \frac{(X^* - \mu)}{\sigma \cdot \sqrt{n}} \quad (13)$$

The single effect of all studied parameters as well as their interactions (cross effects) up through order 3 can be discussed from the Pareto chart illustrated by Fig. 8. The numerical estimates of the effects indicate that the heating temperature (C) has the largest effect on the difference of temperature between both regenerator sides. The charge pressure (B) and the cooling water flow rate has a significant effect too. The operation time and all considered interactions do not have influence on the studied response. Certainly, the Pareto chart can detect the meaningful factors and interactions, but it cannot determine the significance of quadratic interactions (A^2 , B^2 , C^2 and D^2).

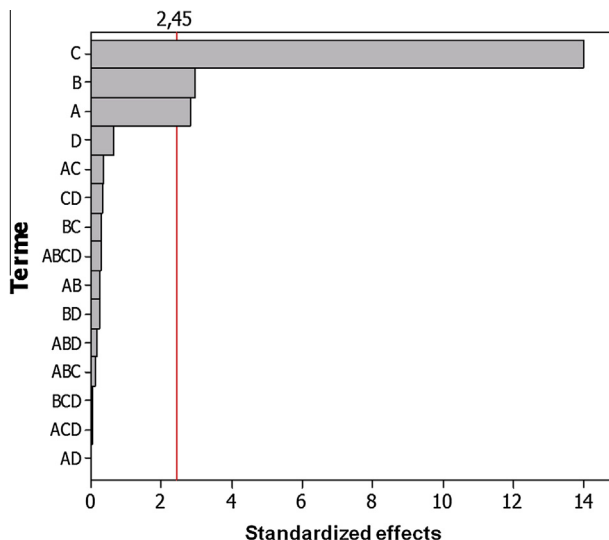


Fig. 8. Pareto chart. A: cooling water flowrate, B: initial pressure, C: heating temperature and D: operation time.

Table 4

Critical probability test.

Variables	Critical probability P	Decision significance with an error of 5% ?
b_0	0.000	(H ₁) Yes
A	0.000	(H ₁) Yes
B	0.000	(H ₁) Yes
C	0.000	(H ₁) Yes
D	0.137	(H ₃) Yes
A^2	0.011	(H ₂) Unknown
B^2	0.000	(H ₁) Yes
C^2	0.062	(H ₃) No
D^2	0.005	(H ₃) Unknown
AB	0.786	(H ₃) No
AC	0.689	(H ₃) No
AD	0.997	(H ₃) No
BC	0.737	(H ₃) No
BD	0.796	(H ₃) No
CD	0.699	(H ₃) No

Table 5

Student test for factors and interactions.

Variables	T-Student	T-Student critique	Decision
A^2	-2.894	2.131	Significant
D^2	-3.325		Significant

To determine the influence of quadratic interactions (A^2 , B^2 , C^2 and D^2) on the studied phenomenon, the critical probability test is used. It is called degree of significance too. It indicates the percentage of risk take when choosing a sample. Each sample is compared to the error value (5%). To make a decision on the significance of a factor, three hypotheses are proposed.

H₁. $P < 0.01 \rightarrow$ the factor is significant for the response.

H₂. $0.01 < P < 0.05 \rightarrow$ Unknown result, must use the Student and the Fisher tests.

H₃. $P > 0.05 \rightarrow$ the factor is not significant for the response.

Table 4 depicted all variables and their corresponding decisions.

We found that the test of the critical probability is not sufficient to judge the significance of the variables A^2 and D^2 . So, we'll go to the Student test to assess the significance of these two variables.

$$t_j = \left| \frac{\hat{\beta}_j}{\sqrt{\hat{\sigma}^2 W_j}} \right| > t(\alpha, n - q - 1) \quad (14)$$

The following table (Table 5) summarizes the values of Student variables, and the decision for each of them. The t critical is determined from statistical table found in literature.

Data were analyzed using multiple regression analysis. So, with consideration of the significant variables for the response: asymmetry of temperature between both regenerator sides, the empirical model as function of coded variables can be written as:

$$Y_{mod} = 32.1476 - 1.3258A + 1.0817B + 4.595C - 0.6286A^2 - 1.0161B^2 - 0.7224D^2 \quad (15)$$

With consideration of Table 2 and the Eq. (2), the prediction model function of uncoded model is writing as follows:

$$Y_{mod}^* = -38.4467 + 0.8276\dot{m} + 7.998P_i + 0.0919T_H - 0.2227\dot{m}^2 - 0.6503P_i^2 - 0.0451T_{op}^2 \quad (16)$$

5.2. Regression test

The coefficient of multiple determinations R^2 and the adjusted coefficient of multiple determinations R_{ad}^2 are respectively of 0.9721 and 0.9442. Both coefficients are statistically significant for the studied response. It suggests that the estimated regression equations for the Case Studied fit the data well. Predicted and experimental results are represented in Fig. 9. It can be seen that all points cluster around the line $y = x$. This change gives an idea of the obtained predicted model and shows that it is in good agreement with the experimental values.

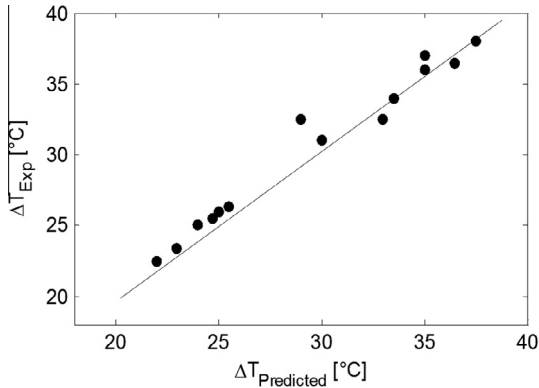


Fig. 9. Normal probability plot of residuals for dissymmetry of temperature between both regenerator sides.

5.3. Iso-surface response

A technique used to help visualize the shape of the three-dimensional response surface is to plot the contours of the iso-surface response. In a plot contour, lines or curves of constant response values are drawn on a graph or plane whose coordinate axes represent the levels of the factors. The contour plot can be used to visualize the response surface behavior function of two factors at a time. Four factors are studied, so for every plot two factors will be held at a constant level and plotting the other two factors. The iso-response surfaces for the factors (the heating temperature, cooling flow and initial charging pressure) influencing the asymmetry of temperature in the regenerator are plotted. They have been plotted in pairs. Fig. 10A. indicates that how variables, heating temperature and initial filling pressure, are related to the studied response while the other factors, cooling water flow rates and operation time are held constant at the highest level (+2). The difference of temperature between both regenerator sides is at its highest value (greater than 30 °C) at the darkest region of the graph. For low heating temperatures this difference of temperature is low (<20 °C) for all variations of the initial charge pressure.

For a heating temperature of 350 °C (level –1), we keep the same level of gap of temperature between both regenerator side, but beyond this level this difference increases significantly, reaching a maximum about 30 °C for a heating temperature of 500 °C (level +2). For low cooling water flow rate (Fig. 10B), the difference of temperature between both regenerator sides increases of approximately 35 °C for a heating temperature of 500 °C (level +2). The increase of cooling water flowrate decreases the difference of temperature between both regenerator sides. It can be noted in

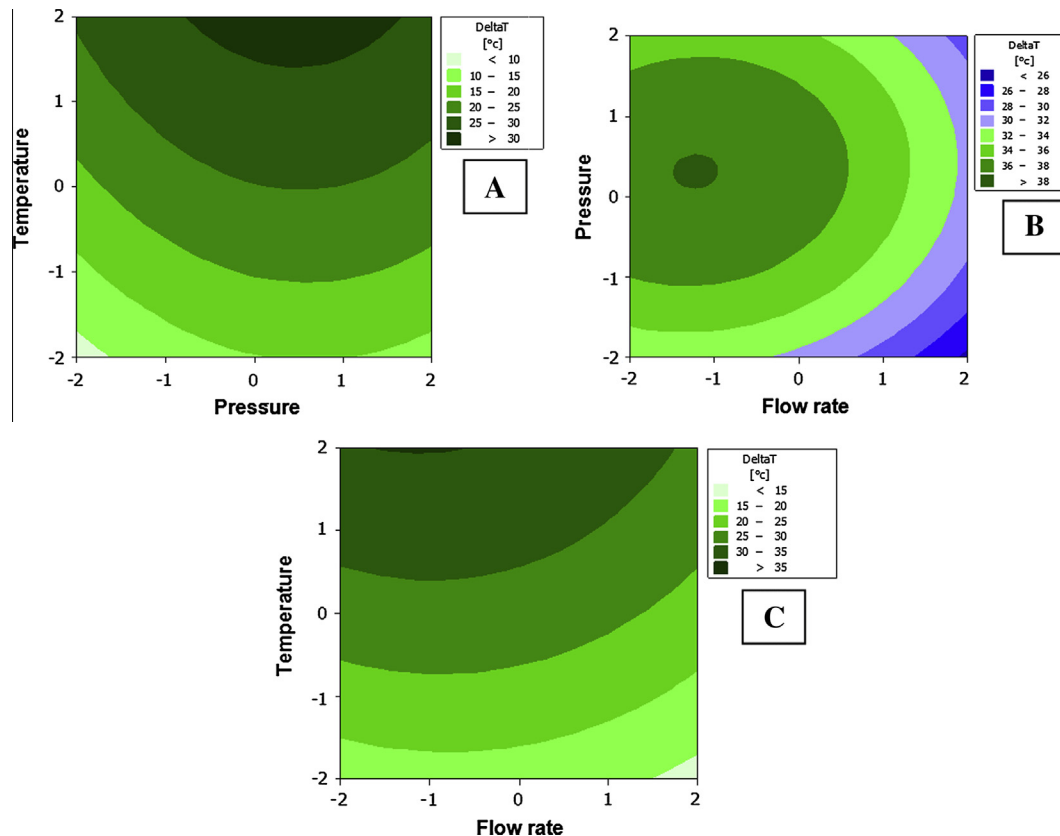


Fig. 10. Contour plot of the difference of temperature between both regenerator sides.

Table 6

Operation parameters for minimal difference of temperature.

Factors	Uncoded value
Heating temperature	300 °C
Initial filling pressure	3 bar
Cooling water flow rate	8.16 l/min
Operation time	20 min

Fig. 9C that regardless of the cooling water flowrate, the difference of temperature between both regenerator sides is very high. The increased of cooling water flow rates from the lowest level (−2) to its highest level (+2), causes the decrease of the temperature difference of 12 °C.

From the predicted model (Eq. (16)), it is possible to determine the optimum parameters minimizing the asymmetry of heat transfer between both regenerator sides. It was found that a minimal difference of temperature can be obtained for the operation parameters depicted in Table 6.

The asymmetry of temperature in a given section of the regenerator is the consequence of air recirculation zones at the outlet or at the inlet of the porous matrix. It causes overlap (friction) between the molecules of air at different temperatures, and therefore at different speeds. Consequently, a frictional drag force appears causing the dissipation of mechanical energy on heat. A difference of density between the particles of warm and cold working fluid causes a pressure drop in the Stirling engine. The pressure drop causes deterioration of heat exchange in the engine, and hence a degradation of performance of the engine. Therefore, stationary vortexes appear at the entrance or the outlet of the regenerator. These vortexes cause the slowdown of the flow in some areas inside the porous matrix.

6. Multi-objectif optimization

The influence of the studied parameters on the engine performances with consideration of the dissymmetry of temperature between both regenerator sides is the subject of a multi-objectif optimization. The goal of this optimization is to minimize the dissymmetry of temperature and to maximize the engine brake power. The Optimization Criteria are depicted in Table 7.

The superposed surfaces are a superposition of the iso-surfaces of both studied responses. The common part (white zone) visualize

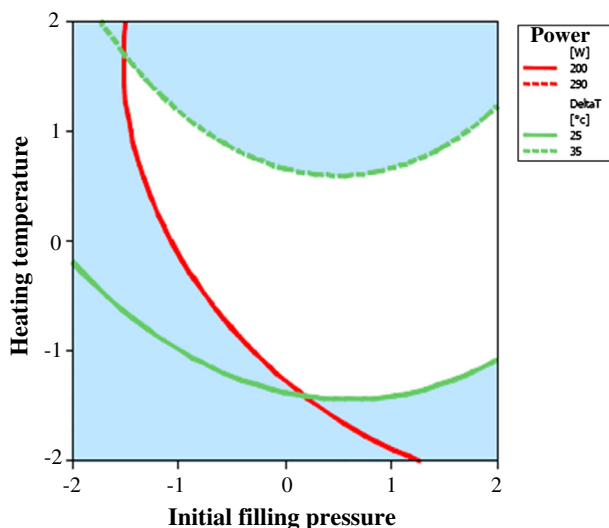


Fig. 11. Superposed surface for both studied responses function of initial filling pressure and heating temperature.

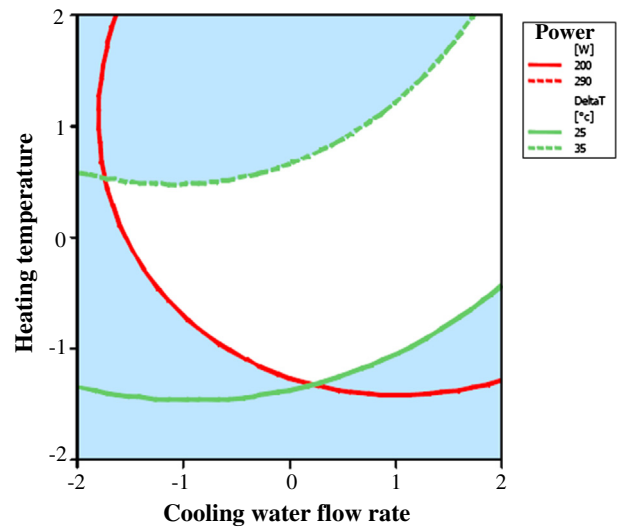


Fig. 12. Superposed surface for both studied responses function of heating temperature and cooling water flow rates.

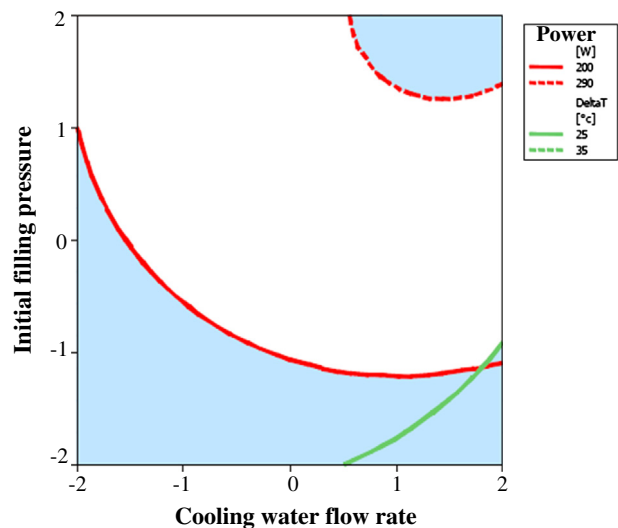


Fig. 13. Superposed surface for both studied responses function of initial filling pressure and cooling water flow rates.

the maximal obtained power and the minimal obtained difference of temperature with the variations of a couple of operation parameters (heating temperature and initial filling pressure (Fig. 11), heating temperature and cooling water flow rates (Fig. 12) and initial filling pressure and cooling water flow rates (Fig. 13).

It can be seen (Fig. 11) that a maximal initial filling pressure associated to an average value of the heating temperature gives a brake power between 200 W and 290 W and a difference of temperature between both regenerator sides between 25 °C and 35 °C.

It can be seen in Fig. 12, that the cooling water flow rates and the heating temperature has a significant influence on both studied responses. A maximal flow rates associated to mean temperature (400 °C) gives the prompted results (choosing in Table 6).

A maximal initial filling pressure associated to a maximal cooling water flow rates gives a maximal brake power. This two operation parameters are not very influent for the difference of temperature between both regenerator sides so for this maximal parameters we obtain a low difference of temperature.

Table 7
Optimization criteria of the two responses.

	Difference of temperature between both regenerator sides (°C)	Mechanical brake power for the Gamma Stirling engine (W)
Objectif	Minimise	Maximise
Minimal value	20	200
Maximal value	25	290
Cible	20	300
Importance	1	2

With consideration of the optimization criteria summarized in Table 7, the optimal functioning parameters which give the maximal brake power (290 W) and the minimal difference of temperature between both regenerator sides (20 °C) are the following:

- Cooling water flow rate: 8.1 l/min.
- Initial filling pressure: 8 bar.
- Heating temperature: 350 °C.
- Operation time: 7 min.

A comparison with the results summarized in Table 6, show that the multi-objectif optimization has changed the initial optimal parameters (Table 6) in order to satisfy all imposed criteria (Table 7).

7. Concluding remarks

In this paper, a new phenomenon causing the thermal energy dissipation in a Gamma type Stirling engine was studied. The asymmetry of temperature between both regenerator sides consumes a part of the exchanged thermal energy. The non-symmetrical design of Gamma Stirling engine is the principal causes of this phenomenon. An experimental design was performed to determine the influence of some operation parameters on the observed phenomenon. The heating temperature is the most significant parameter. An increase of 200 °C of the heating temperature increases the difference of temperature between both regenerator sides of 20 °C. An analytical model linking the asymmetry of temperature to influent factors and interactions was proposed. With modeling, optimal operation parameters can be established. A minimal difference of temperature between both regenerator sides of 17 °C can be obtained by setting a minimal heating temperature and filling pressure and maximal cooling water flow rate.

Second experimental design was proposed for a multi-objective optimization. The choosing Criteria are to maximize the engine power and to minimize the difference of temperature between both regenerator sides. A maximal power of 260 W and a difference temperature of 20 °C are the optimal obtained responses.

A geometric solution can be proposed to minimize the dissymmetry of temperature between both regenerator sides. The alimentation of engine cooler by the cooling water is actually made in only one side. A new cooler will be manufactured with alimentation in his both sides. The new configuration will ensure a homogeneous distribution of working fluid temperature at input of the regenerator.

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References

- [1] Hsu ST, Lin FY, Chiou JS. Heat-transfer aspects of Stirling power generation using incinerator waste energy. *Renewable Energy* 2003;28:59–69.
- [2] Popescu G, Radcenco V, Costea M, Feidt M. Optimisation thermodynamique en temps fini du moteur de Stirling endo- et exo-irréversible. *Rev Gén Therm* 1996;35:656–61.
- [3] Tilili I, Timoumi Y, Ben Nasrallah S. Numerical simulation and losses analysis in a Stirling engine. *Int J Heat Technol* 2005;10.
- [4] Walker G. *Stirling Engines*. Oxford: Clarendon Press; 1980.
- [5] Halit K, Huseyin S, Atilla K. Manufacturing and testing of a V-type Stirling engine. *Turk J Engine Environ Sci* 2000;24:71–80.
- [6] Martaj N, Grosu L. Exergetical analysis and design optimization of the Stirling engine. *Int J Exergy* 2006;3(1):45–66.
- [7] Kolin I. *The Stirling motor: history-theory-practice*. Dubrovnik: International University Center; 1991.
- [8] De Vos A. *Endoreversible thermodynamics for solar energy conversion*. Oxford: Oxford Science Publishers; 1992.
- [9] Iwamoto S, Hirata K. Comparison of low and high temperature differential Stirling engine. *Proceeding 31st IECEC* 1996;2:1259–64.
- [10] Gheith R, Aloui F, Tazerout M, Ben Nasrallah S. Experimental investigations of a gamma Stirling engine. *Int J Energy Res* 2012;36(2):1175–82.
- [11] Tilili I, Timoumi Y, Ben Nasrallah S. Numerical simulation and losses analysis in a Stirling engine. *Int J Heat Technol* 2006;24:96–102.
- [12] Ahmadi Mohammad H, Mohammadi Amir H, Dehghani Saeed, Barranco-Jiménez Marco A. Multi-objective thermodynamic-based optimization of output power of Solar Dish-Stirling engine by implementing an evolutionary algorithm. *Energy Convers Manage* 2013;75:438–45.
- [13] Chen WL, Wong KL, Po LW. A numerical analysis on the performance of a pressurized twin power piston gamma-type Stirling engine. *Energy Convers Manage* 2012;62:84–92.
- [14] Cheng CH, Yang H-S, Keong L. Theoretical and experimental study of a 300-W beta-type Stirling Engine. *Energy* 2013;59:590–9.
- [15] Kato Y, Baba K. Empirical estimation of regenerator efficiency for a low temperature differential Stirling engine. *Renewable Energy* 2014;62:285–92.
- [16] Cheng C-H, Yang H-S. Optimization of geometrical parameters for Stirling engines based on theoretical analysis. *Appl Energy* 2012;92:395–405.
- [17] Glushenkov M, Sprenkeler M, Kronberg A, Kirillov V. Single-piston alternative to Stirling engines. *Appl Energy* 2012;97:743–8.
- [18] Ahmadi Mohammad Hossein, Sayyaadi Hoseyn, Dehghani Saeed, Hosseinzade Hadi. Designing a solar powered Stirling heat engine based on multiple criteria: maximized thermal efficiency and power. *Energy Convers Manage* 2013;75:282–91.
- [19] Kato Yoshitaka, Baba Kazunari. Empirical estimation of regenerator efficiency for a low temperature differential Stirling engine. *Renewable Energy* 2014;62:285–92.
- [20] Chen Wen-Lih, Wong King-Leung, Chen Hung-En. An experimental study on the performance of the moving regenerator for a c-type twin power piston Stirling engine. *Energy Convers Manage* 2014;77:118–28.
- [21] Formosa F, Despesse G. Analytical model for Stirling cycle machine design. *Energy Conversion Manage* 2010;51:1855–63.
- [22] Chen Wen-Lih, Wong King-Leung, Chen Hung-En. An experimental study on the performance of the moving regenerator for a c-type twin power piston Stirling engine Wen-Lih Chen. *Energy Convers Manage* 2014;77:118–28.
- [23] Andersen SK, Carlsen H, Thomsen PG. Numerical study on optimal Stirling regenerator matrix designs taking into account the effects of matrix temperature oscillations. *Energy Convers Manage* 2006;47:894–908.
- [24] Gheith R, Aloui F, Ben Nasrallah S. Study of the regenerator constituting material influence on a gamma type Stirling engine. *J Mech Sci Technol* 2012;26(4):1251–5.
- [25] Tavakolpour AR, Zomorodian A, Golneshan AA. Simulation, construction and testing of a two-cylinder solar Stirling engine powered by a flat-plate solar collector without regenerator. *Renew Energy* 2008;33:77–87.
- [26] Iwamoto S, Toda F, Hirata K, Takeuchi M. Comparison of low and high temperature differential Stirling engines. *Proc Eighth Int Stirling Engine Conf* 1997:29–38.
- [27] Formosa F, Despesse G. Analytical model for Stirling cycle machine design. *Energy Convers Manage* 2010;51:1855–63.
- [28] Karabulut H, Çınar C, Öztürk E, Yücesu HS. Torque and power characteristics of a helium charged Stirling engine with a lever controlled displacer driving mechanism. *Renewable Energy* 2010;35:138–43.
- [29] Cinar C, Yücesu S, Topgul T, Okur M. Beta-type Stirling engine operating at atmospheric pressure. *Appl Energy* 2005;81:351–7.
- [30] Chen J, Yan Z, Chan L, Andersen B. Efficiency bound of a solar-driven Stirling heat engine system. *Int J Energy Res* 1997;25:805–12.
- [31] Cleber Z, Minku LL, Lundermir TB. Design of experiments in Neuro-Fuzzy systems. *Int J Comput Intell Appl* 2010;9(2):137–52.
- [32] Pillet M. *Introduction aux plans d'expériences par la méthode Taguchi*. Les éditions d'organisations, ISBN 2-7081-1442-5, 1994.
- [33] La Goupie J. *Méthode des plans d'expériences*. Paris: Imprimerie Gauthier-Villars; 1993.
- [34] Design of Experiments for Engineers and Scientists. Butterworth-Heinemann; 2003, ISBN 0750647094.
- [35] Hayajneh MT, Tahat MS, Bluhm J. A study of the effects of machining parameters on the surface roughness in the end-milling process. *J Mech Ind Eng* 2007;1:1–5.
- [36] Lenth RV. Quick and easy analysis of unreplicated factorials. *Technometrics* 1989;31:469–73.